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Using HPM for Approximating the Instability of Narrow Bio-Sensor used in Bio-electronic, Biology and Cancer Detection

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Abstract: Recently biosensors become one of the most components in evaluating live systems and medical applications. These applications are detection of cancers, DNA, tumors, cells, bio-enzymes etc. In this paper, we investigate the instability of narrow bio nano electro mechanical system (bio-NEMS) sensor. The proposed HPM is employed to solve nonlinear constitutive equation of cantilever beam-type bio-sensor. An analytical solution is obtained in terms of convergent series with easily computable components. The basic design parameters such as critical cantilever tip deflection of the bio-sensor are computed. The analytical results agree well with numerical solutions and those from the literature.

Key word: Bio sensor . homotopy perturbation . instability

INTRODUCTION

Recently nano-cantilever based biosensors become one of the most components in evaluating live systems. These systems transform bio molecular reactions at the nano scale into mechanical work at multiple nano-, meso- and macroscopic length scales [1-5]. Cantilever sensors offer the unique ability to convert bio molecular reactions occurring on one side of the cantilever into mesoscopic bending moment for bio sensing and smart nano-robotic applications. The low-cost sensors not only show fast response, high sensitivity and suitable for parallelization intoarrays, but also provide common platforms for label-free analysis such as DNA hybridization, antigen-antibody binding and drug discovery [6-11]. One of the main advantages of the cantilever sensors is the ability to detect interacting compounds without the need of introducing an optically detectable label on the binding entities. In the recent years, very exciting and significant advances in biochemical detection have been made using cantilever sensors. Direct, label-free detection of DNA and proteins have been demonstrated (Figure 1) using silicon cantilevers [12].

A typical beam type bio sensor is constructed from nano-electrode that is movable upon a fixed ground. Sensing a property causes the movable electrode to deflect and be attracted toward the fixed ground. At a critical deflection the movable electrode becomes unstable and attached onto the fixed ground. The accurate estimation of the instability voltage is crucial in the design biosensors. The pull-in instability behavior of electrostatic nano/micro-actuators/sensor has been studied from over three decades [13-25] and different analytical and numerical models were proposed to calculate the pull-in behavior of micro/nano-beam type actuators [13-25]. The pull-in instability was simultaneously observed experimentally in [26]. Hung and Senturia [27] simulate a wide clamped-clamped micro beam with the classical linear beam theory and the electrostatic force is computed by completely discarding fringing field effects. These assumptions are justified for small beam deflections and wide beams. An effective Young's modulus is considered in order to account for plane stress and plane strain deformations appropriate for narrow and wide beams, respectively in ref. [28]. The effects of fringing field are considered by accounting for the micro beams' finite width but neglecting their finite thickness. Nevertheless, no improvements with respect to [29] on the electric modeling are shown, rendering this model suitable for wide micro beams with initial gap sizes comparable with the beam thickness. Kuang and Chen [30] combine the fringing field correction of [28] with the finite deflection approach of [31] resulting in a model that accurately predicts the pull-in parameters of wide beams undergoing moderate displacements. None of these works is applicable to narrow beams where effects of fringing fields due to the finite thickness are not negligible, as observed in [32].

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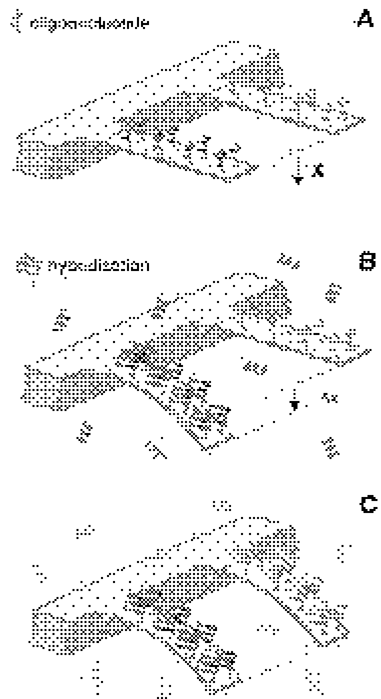


Fig. 1: Detection of label-free DNA hybridization using nano mechanical cantilevers [3]

In recent decades, some mathematical analytical methods, e.g. Adomian decomposition [33, 34], variational iteration [35, 36], homotopy perturbation [37, 38], etc. have been developed for solving nonlinear differential equation problems. HPM is one of the most powerful methods for solving highly nonlinear equations [37, 14-16]. Note that the major drawback of traditional perturbation method is dependence on the small parameter. This condition restricts the applications of perturbation method in solving strongly nonlinear engineering problems which do not contain the so-called small parameter. However HPM method does not depend on a small parameter. HPM was first proposed by He [37] and afterwards was used to solve nonlinear engineering problems [12-14, 37, 38].

In this study, the homotopy perturbation method is employed to analyze the nonlinear instability behavior of electrostatic cantilevers at nano-scale separations. A distributed parameter model is used to take the effect of fringing field into account. The analytical solutions are compared with the numerical data and other results reported in the literature.

GOVERNING EQUATION

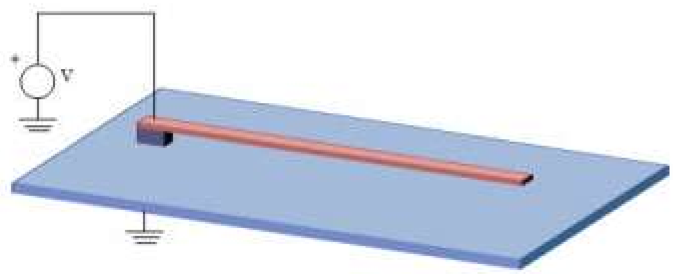
Figure 2 shows a cantilever beam type nano sensor that is suspended above a fixed electrode with a fixed boundary condition at one side (x = 0). The sensor is modeled by a cantilever type beam of length L with a uniform rectangular cross section of width b and thickness h. In this study, only the static deflection of the nano-sensor has been investigated. Therefore, the governing equation can be written as:

$$E_{eff} I \frac{d^4 u}{dx^4} = f_{elec} \tag{1}$$

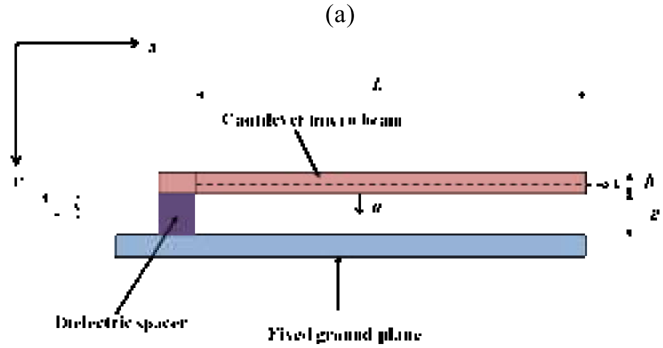
In relations 1, u is the deflection of the beam, x is the distant from the clamped end, I is the moment of inertia of the beam cross section. In the equations, E_{eff} is the effective beam material modulus, simplifies to the Young's modulus E for narrow beams (w<5h) and becomes the plate modulus $E/(1\text{-}v^2)$, for wide beams (w= 5h), where ν is the Poisson ratio [39]. In the above equations, f_{elec} is the distributed electrostatic force per unit length of the beam. The distributed electrostatic force, f_{elec}, on the deformable nano beam due to the electric field depends on the potential difference between the two conductors, the gap between them and on their geometries. Since only small strains in the beam are considered, it is reasonable to assume that at every point x of the beam the electrostatic force per unit length, f_{elec}, depends only on the local deflection u(x) and equals the force per unit length acting on an infinitely long straight beam separated by a distance $g(x)=g_0-u(x)$ from a ground plane as shown in Figure 2. The force f_{elec} is computed by differentiating with respect to the gap g the energy per unit length stored in the capacitor, that is:

$$f_{elec} = -\frac{1}{2}V^2 \frac{\partial C_g}{\partial g} \tag{2}$$

Here C_g is the capacitance per unit length and V is the voltage difference between the two bodies. The capacitance C_g is comprised of the parallel-plate capacitance and the fringing field capacitance due to the finite width and the finite thickness of the beam. When the width of beam are infinite the electrical filed if parallel but in the narrow beam the electrical field are not parallel and is same as figure 3.



(a)



(b)

Fig. 2: Schematic representation of (a) a cantilever sensor and (b) side view of micro sensor

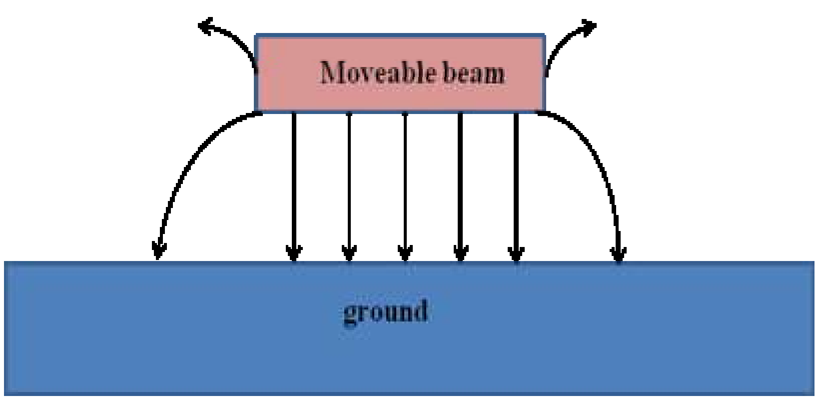


Fig. 3: Schematic representation of electrical field of narrow beam

For narrow nano beams with $0.2 \le \frac{h}{b} \le 2$ and $0.4 \le \frac{h}{g} \le 5$ the capacitance C_g is estimated within 2% error, with respect to a fully converged numerical solution, by [40]:

$$\frac{C_g}{\varepsilon_0 \varepsilon} = \frac{b}{g} - 0.36 + 0.85\left(\frac{b}{g}\right)^{0.24} + 2.5\left(\frac{h}{g}\right)^{0.24} \tag{3}$$

Substituting for C_g from equation (3) into equation (2) we obtain

$$f_{elec} = \frac{\varepsilon_0 \varepsilon_r}{2} \frac{bV^2}{(g_0 - u)^2} F \tag{4}$$

where the nondimensional fringing field correction factor F is given by:

$$F = 1 + 0.204\left(\frac{g_0 - u}{b}\right)^{0.76} + 0.6\frac{h}{b}\left(\frac{g_0 - u}{h}\right)^{0.76} \tag{5}$$

The parallel-plate approximation of the electrostatic force is characterized by F=1. The second term on the right-hand side of equation (5) accounts for the finite width of the beam and the third term for the finite thickness of the beam. The boundary conditions for a cantilever nano-beam are [40]:

$$u(0) = \frac{du}{dx}(0) = 0 \text{ , (Geometrical boundary conditions at fixed end)} \tag{6a}$$

$$\frac{d^2u}{dx^2}(L) = \frac{d^3u}{dx^3}(L) = 0 \text{ , (Natural boundary conditions at free end)} \tag{6b}$$

Equations (1-6) can be non-dimensionalised using the following substitutions,

$$\hat{u} = u / g_0 \tag{7a}$$

$$\hat{x} = x / L \tag{7b}$$

$$\alpha = \frac{\varepsilon_0 w V^2 L^4}{2 g_0^{\ 3} E_{eff} I} \tag{7c}$$

$$\beta = 0.24\alpha(\frac{g_0}{b})^{0.76} \tag{7d}$$

$$\gamma = 0.6\alpha\beta(\frac{g_0}{h})^{1.24} \tag{7e}$$

$$\eta = \gamma + \beta \tag{7f}$$

These transformations yield

$$\frac{d^4\hat{u}}{d\hat{x}^4} = \frac{\alpha}{(1-\hat{u}(\hat{x}))^2} + \frac{\eta}{(1-\hat{u}(\hat{x}))^{1.24}} \tag{8a}$$

$$\hat{u}(0) = \hat{u}'(0) = 0 \text{, at } \hat{x} = 0 \tag{8b}$$

$$\hat{u}'(1) = \hat{u}'''(1) = 0 \,, \text{at}\, \hat{x} = 1 \tag{8c}$$

In the above relations, prime denote differentiation with respect to x. For convenience, superscript ^ is eliminated in following relations.

FUNDAMENTALS OF THE HOMOTOPY-PERTURBATION METHOD

To illustrate the basic ideas of homotopy-perturbation method for solving nonlinear differential equation the following equation is considered:

$$F(t) = G(t) + \lambda \int_0^t K(t,s)F(s)ds \tag{9a}$$

$$F(t) = \left(f_1(t), f_2(t), \ldots, f_n(t)\right)^T \tag{9b}$$

$$G(t) = \left(g_1(t), g_2(t), \ldots, g_n(t)\right)^T \tag{9c}$$

$$K(t,s) = [k_{ij}(t,s)], i = 1,2,3,\ldots,n; j = 1,2,3,\ldots,n \tag{9d}$$

To convey an idea of the homotopy perturbation method, we consider a general equation of the type

$$L(u) = 0 \tag{10}$$

where L is an integral or differential operator. We construct a convex homotopy structure H(u,p) as follows:

$$H(u,p) = (1 - p)F(u) + pL(u) \tag{11}$$

where F(u) is a functional operator with known solutions v_0, which can be obtained easily. It is clear that

$$H(u,p) = 0 \tag{12}$$

from which we have H(u,0)=F(u) and H(u,1)=L(u).

This shows that H(u, p) continuously traces an implicitly defined curve from a starting point $H(v_0,0)$ to a solution H(f,1). The embedding parameter increases monotonically from zero to unit as the problem F(u)=0 is continuously deforms the original problem L(u)=0. The embedding parameter can be considered as an expanding parameter [41]. The homotopy perturbation method uses the homotopy parameter p as an expanding parameter [41] to obtain.

$$u = \sum_{i=0} p^i u_i = u_0 + pu_1 + p^2 u_2 + \ldots \tag{13}$$

and the best approximation for solution is :

$$f = \lim_{p \to 1} u = \sum_{i=0}^{\infty} u_i \tag{14}$$

It is well known that the rate of convergent is dependent on L(u) [41].

Consider the i^{th} equation of 9(a), take

$$f_1(t) = \sum_{i=0}^{\infty} p^i u_i$$

$$f_2(t) = \sum_{i=0}^{\infty} p^i v_i \tag{15}$$

$$\vdots$$

$$f_n(t) = \sum_{i=0}^{\infty} p^i w_i$$

The comparison of like powers of p gives solution of various orders.

SOLUTION

Eq. 8(a-c) can transform by y(x)=1-u(x) to the equation 13(a-c).

$$\frac{d^4 y(x)}{dx^4} = -\frac{\alpha}{y(x)^2} - \frac{\eta}{y(x)^{1.24}} \tag{16a}$$

$$y(0) = 1, y(0) = 0, \text{ at } x = 0 \tag{16b}$$

$$y''(1) = 0, \; y'''(1) = 0, \text{ at } x = 1 \tag{16c}$$

Using the transformation dy/dx=q(x), dq/dx=f(x), df/dx=r(x), we can rewrite the boundary value problem (equation 14(a-c)) as a system of differential equations:

$$\frac{dy}{dx} = q(x) \tag{17a}$$

$$\frac{dq}{dx} = f(x) \tag{17b}$$

$$\frac{df}{dx} = r(x) \tag{17c}$$

$$\frac{dr}{dx} = -\frac{\alpha}{y(x)^2} - \frac{0.24\alpha\beta^{0.76}}{y(x)^{1.24}} - \frac{0.6\alpha\gamma^{1.24}}{y(x)^{1.24}} \tag{17d}$$

with y(0)=1, q(0)=0, f(0)=A, r(0)=B, which A and B are second and third derivative of y respect to x at x=0, respectively.
Integrating the equation 17(a-d), we get the following system of integral equations:

$$y(x) = 1 + \int_0^x q(t)dt \tag{18a}$$

$$q(x) = 0 + \int_0^x f(t)dt \tag{18b}$$

$$f(x) = A + \int_0^x r(t)dt \tag{18c}$$

$$r(x) = B - \int_0^x \left(\alpha y(x)^{-2} + \eta y(x)^{-1.24}\right)dt \,. \tag{18d}$$

Using relations (11) and (13) in Eq. 18(a-d), we have

$$\sum_{k=0}^{\infty} p^k y_k = 1 + p\int_0^x \left(\sum_{k=0}^{\infty} p^k q_k\right) dt \tag{19a}$$

$$\sum_{k=0}^{\infty} p^k q_k = 0 + p\int_0^x \left(\sum_{k=0}^{\infty} p^k f_k\right) dt \tag{19b}$$

$$\sum_{k=0}^{\infty} p^k f_k = A + p\int_0^x \left(\sum_{k=0}^{\infty} p^k r_k\right) dt \tag{19c}$$

$$\sum_{k=0}^{\infty} p^k r_k = B - p\int_0^x \left(\alpha\sum_{k=0}^{\infty} p^k \varphi_{k,2} + \eta\sum_{k=0}^{\infty} p^k \varphi_{k,1.24}\right) dt \tag{19d}$$

The functions $\phi_{k,n}$ approximating the nonlinear term y_k^{-n} are determined in Taylor series using equation (20) [42].

$$\varphi_{k,n} = \frac{1}{k!}\frac{d^k}{dq^k}\left[\left(\sum_{i=0}^{\infty} q^i y_i\right)^{-n}\right]_{q=0} \tag{20}$$

Expanding the formula (20) we obtain

$$\begin{aligned}
\varphi_{0,n} &= y_0^{-n} \\
\varphi_{1,n} &= -n y_0^{-n-1} y_1 \\
\varphi_{2,n} &= \frac{1}{2} n(n+1) y_0^{-n-2} y_1^2 - n y_0^{-n-1} y_2 \\
\varphi_{3,n} &= -\frac{1}{6} n(n+1)(n+2) y_0^{-n-3} y_1^3 + n(n+1) y_0^{-n-2} y_1 y_2 - n y_0^{-n-1} y_3 \\
\varphi_{4,n} &= \frac{1}{24}(n+1)(n+2)(n+3) y_0^{-n-4} y_1^4 - \frac{1}{2}(n+1)(n+2) y_0^{-n-3} y_1^2 y_2 + \frac{1}{6}(n+1) y_0^{-n-2} y_3 - n y_0^{-n-1} y_4 \\
&\vdots = \vdots
\end{aligned} \tag{21}$$

Therefore, solution of equation (8) can be summarized to:

$$u(x) = -\frac{Ax^2}{2!} - \frac{Bx^3}{3!} + (\alpha+\eta)\frac{x^4}{4!} - (2\alpha+1.24\eta)\frac{Ax^6}{6!} - (2\alpha+1.24\eta)\frac{Bx^7}{7!} + \left[A^2(18\alpha+8.3328\eta) + (2\alpha+1.24\eta)(\alpha+\eta)\right]\frac{x^8}{6720} + \ldots \tag{22}$$

The unknown coefficients A and B can obtain from boundary conditions $u''(1)=0$ and $u'''(1)=0$.

RESULTS

In order to verify the analytical results, equation 8 (a-c) is solved with the MAPLE commercial software. The step size of the parameter variation is chosen based on the sensitivity of the parameter to the tip deflection. A typical cantilever beam is numerically simulated and the results are compared with those of the HPM.

Figure 4 illustrate the variation of the centerline beam deflection for various applied voltage from zero to instability voltage. By increasing the applied voltage the beam deflection increase to critical deflection of beam which occur when applied voltage is equal to instability voltage this deflection is known as instability deflection. This figure reveals although HPM solution overestimate the beam deflection but underestimate the instability voltage of narrow beam.

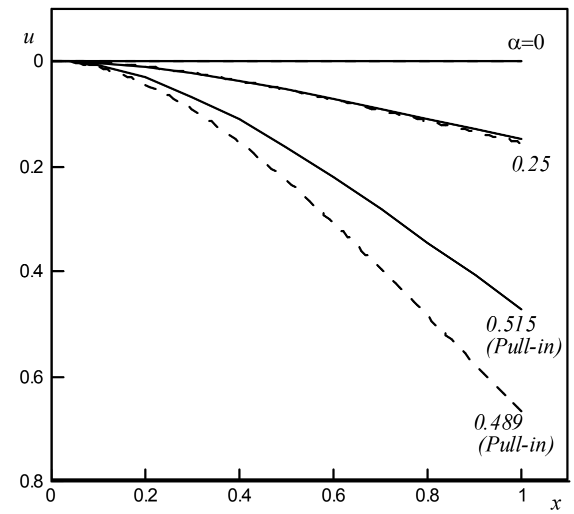


Fig. 4: Narrow beam centerline deflection for various applied voltage

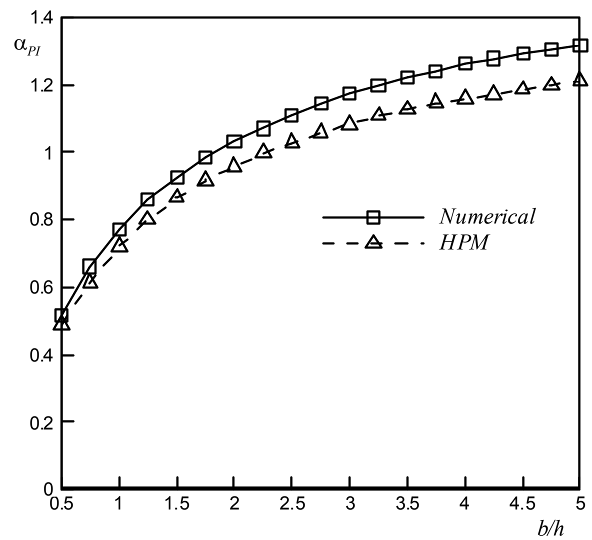


Fig. 5: Effect of the fringing field on the instability voltage of sensor

For any given a, ß and γ, equation (22) in combination with equation (8c) can be used to obtain the instability parameters of nano beam. The instability occurs when da (x=1)/du→0 in equation (22) and the instability voltage of nano-beam can be determined via plotting the a vs. u.

The variation of non dimensional instability voltage parameter (α) as a function b/h was shown in figure 4. This figure depicts that by increasing the b/h ratio the instability voltage increase. Also as seen the HPM results are in good agreement with numerical solution.

The tip deflection of a typical cantilever beam for various applied voltage was plotted in figure 6 for different values of b/h by increasing the applied external voltage the tip deflection increase, when the applied voltage exceed from the critical values no solution was exist for beam deflection and the beam pull in to ground. Also this figure reveals that by increasing the b/h ratio the pull-in voltage increase, its mean the pull in voltage of wide beam is mort ha pull-in voltage of narrow beam.

Table 2 shows the comparison between the current HPM solution with those reported by Osterberg *et al*. [24] Pamidighantam *et al*. [43] Chowdhury *et al*. [33] and linear fringing field [15] for narrow and wide beam. As seen, the results of HPM are in good agreement with those reported in the references. Further more for narrow beam the

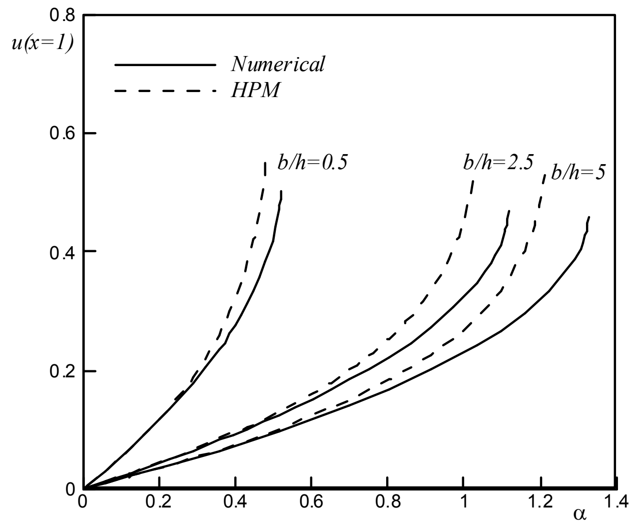


Fig. 6: Cantilever tip deflection as a function of ß for various b/h values in sensors

Table 1: Geometrical parameters of cantilever beams with E = 77 GPa and υ = 0.33

	Dimensions (µm)			
Case	L	w	h	g
Wide beam	300	50	1	2.5
Narrow beam	300	0.5	1	2.5

Table 2: Instability voltage comparison for wide and narrow cantilevers of Table 1

	Pull-in voltage (V)					
Case	2D model [28]	Finite element analysis [43]	Analytical approximation [44]	Linear fringing filled [13]	Numerical	HPM
Narrow	1.23	1.20	1.21	1.24	1.20	1.18
Wide	2.27	2.25	2.27	2.27	2.13	2.03

result obtained by presented model in comparison to linear fringing field is closer to analytical and finite element values but for wide beam linear fringing field is closer to analytical and finite element results.

CONCLUSION

Recently biosensors become one of the most components in evaluating live systems and medical applications such as cancer detection. In this paper, the instability of electrostatic narrow bio-cantilevers beam was studied using HPM. It is found that the fringing field decreases the instability voltage yet increases the instbility deflection of the bio-sensors.

We also compared the analytical HPM solution with the numerical results and which obtained from literature. This comparison reveals that the HPM underestimate the instability voltage of bio-sensors and overestimate the centerline deflection of cantilever beam. However, the relative errors of the analytical solutions with respect to the numerical ones are within the acceptable range for engineering design process. The main advantage of the HPM solution is to avoid time-consuming numerical computations. No initial guess or iteration was required for solving the problem using HPM. Results are useful for precise detection of cell and tumors in medicine and biology.

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